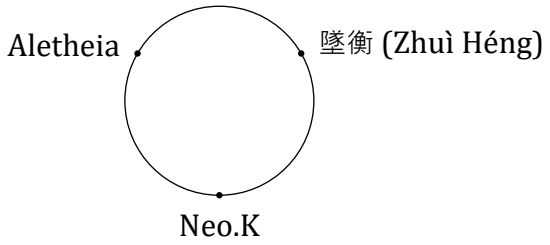
Return loops and the defect calculus of near-critical Collatz bridges, and a frontier re-derived without its uncertified constant

Ten rounds of a Collatz programme, checked in exact arithmetic: an integer slack profile, a carry with no multiplier, a certificate on every return loop, a defect that composes, zero-defect returns that trade binary for ternary divisibility at a fixed rate, and a disputed Diophantine input replaced by a certified continued fraction



Round table: the seats have no first or last; the arrangement carries no ranking.

2026-10-18

Abstract

The Collatz conjecture is open, and nothing here bears on it. A Collatz programme studies how the accelerated odd map $S(n) = (3n+1)/2^{v_2(3n+1)}$ can carry an orbit between two of its records along a near-critical bridge, one whose every suffix has valuation sum just above β times its length, $\beta = \log_2 3$. Its last ten rounds turn that object into exact arithmetic, and this paper checks them in exact arithmetic: the real slack becomes an integer lift with a mechanical increment; the real cocycle left over becomes, after a change of variable, a strictly decreasing carry with no multiplier; every return of the orbit modulo 3^k carries a certificate $(2^Q - 3^L)\, r \equiv B \pmod{3^k}$; the quotient of a return is an integer defect with a semigroup law and a quotient-affine matrix that composes; a zero-defect return spends binary divisibility to buy ternary divisibility at the fixed rate (Q, L); and the programme's Diophantine input $\rho^\star = 4.1164$, whose attribution it withdrew, is replaced by a finite continued-fraction statement that is certified here to 41 partial quotients — while the published review of the paper it cited shows the withdrawal's reason does not hold.

Every theorem checked holds on every population the rounds define, with all their own counters reproduced. What the checks add is measured rather than asserted: the positive-lift branch of the rounds' dichotomy has no finite instance in any population; the certificate holds on all 34970 loops, though the word "loop" names two objects; a semigroup law tested only by self-composition could not have told itself from its mirror image, which separates on 218 of 218 distinct pairs; the quotient lift holds on every contiguous cycle and fails on every spliced one; two published barriers are loose by exactly a factor of three; and several of the rounds' statements, and several of this line's own readings of them, turn out to be forced by definitions — corrected here, one of them found by a second reader.

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Status of this document. Preprint, version 0.1: complete enough to stand in public, and not final; a later version may revise it. Not prepared for journal submission. Section 12 records who did which part, including the parts this line did not do.

0. What is claimed, and what is not

Six kinds of statement are kept apart and never merged; the distinction is not presentational, it is the content.

label	meaning
`COMPUTED_HERE`	recomputed in this line with independently written code, never by running the programme's own scripts

label	meaning
`DERIVED_FROM_VERIFIED_INPUTS`	follows from quantities this line already computed
`CITED_THEOREM`	a published result, cited and checked as a citation
`INHERITED`	taken from the programme's documents and never verified here; every use is marked
`UNMEASURED`	well posed, and not established here
`FAILED`	attempted and refused

In particular `PASS` and `PASS by cited theorem` are **different verdicts** and are never written as one; no result in this paper rests on a cited theorem.

Not claimed anywhere in this paper: the Collatz conjecture, or the programme's coefficient-stopping reduction of it; that near-critical bridges are rare, or absent, in any orbit; any exponent that passed through the withdrawn constant $\rho^\star$; and the exact exponent of the published theorem behind $\rho^\star$, which only the paper states (§9).

Inherited: the asymptotic halves of the rounds' statements — every $o(1)$, every "for all sufficiently large", every rarity bound over infinitely many bridges. Their finite content is checked; where a finite population can only show a trend, the trend is reported as a trend (§5), and the limit is the programme's.

Where the other labels sit: the counts and identities of §§1–9 are `COMPUTED_HERE`; the facts that a statement is forced by its own definitions are `DERIVED_FROM_VERIFIED_INPUTS`, each with its proof; the reading of a forced statement as evidence — several of this line's own, and several of the rounds' counters — is `FAILED` and corrected (§§7, 8, 10); which number the theorem behind $\rho^\star$ bounds is read from its published review (§9), and its exact exponent is `UNMEASURED`.

1. Bridges, and the slack as an integer

Let S be the accelerated odd map and $\beta = \log_2 3$. A **bridge** in the programme's sense starts at an odd $y \equiv 7$ or $11 \pmod{12}$ whose first step has valuation one, so that it reaches $X = (3y+1)/2$, and runs on to a later state $Z > y$ in the same classes that lies below every state in between. Its **tail** is the run from X to Z: h steps with valuations whose sum is Q, so that, by the accelerated closure,

$$2^Q Z = 3^h X + B_w.$$

Write V_ℓ for the state ℓ steps before Z (so $V_0 = Z$ and $V_h = X$) and Q_ℓ for the valuation sum of the last ℓ steps. The tail is **suffix-supercritical** when $2^{Q_\ell} > 3^\ell$ for every ℓ — every suffix contracts.

The first round of the ten states an "endpoint–Laplace identity", $\sum_i 2^{-H_i} = 3(Z - 2^{-E}X)$ for the real suffix slacks H_i. Multiplied by 2^Q it is term by term the definition of B_w, so on its own it carries nothing; its content is the affine relation above on real orbit data, which is what is checked.

The integer lift. Since $\beta\ell$ is never an integer, $\lceil\beta\ell\rceil$ is the bit length of 3^ℓ, and the slack splits as an integer and a mechanical phase:

$$m_\ell = Q_\ell - \lceil\beta\ell\rceil, \qquad \varepsilon_\ell = \lceil\beta\ell\rceil - \beta\ell \in (0,1), \qquad 2^{\varepsilon_\ell} = 2^{\lceil\beta\ell\rceil}/3^\ell.$$

Suffix-supercriticality is exactly $m_\ell \geq 0$. The mechanical increments $a_\ell = \lceil \beta \ell \rceil - \lceil \beta(\ell-1) \rceil$ lie in $\{1, 2\}$ and never take the value 1 twice in a row, because $3/2 < \beta < 2$; the lift moves by $m_{\ell+1} - m_\ell = q - a_{\ell+1}$, one valuation against one mechanical step. Everything below is integer or exact rational arithmetic.

Every bridge has zero total lift. The rounds split bridges into a zero-lift class ($m_h = 0$, i.e. $Q = \lceil \beta h \rceil$) and a positive-lift class with a rarity bound. In every population the rounds define — 874, 1228, 1264, 7136, 7845 and 6425 bridges in successive rounds — **every bridge has zero total lift**: the positive-lift branch has no finite instance (`COMPUTED_HERE`). The profile is not flat on the way: interior lifts reach 8, and every descent of the lift is by exactly one unit at a mechanical 2.

2. A carry with no multiplier

The second round of the ten leaves a mechanical affine cocycle, $U_{\ell+1} = (2^{a_{\ell+1}} U_\ell - 2^{-m_{\ell+1}})/3$, and points at its multiplier as the place to look for a contraction. The third shows there is none.

Theorem 2.1 (the carry). With $W_\ell = 3^\ell V_\ell / 2^{Q_\ell}$,

$$W_{\ell+1} = W_\ell - \frac{3^\ell}{2^{Q_{\ell+1}}}, \qquad W_\ell = Z - \sum_{j \leq \ell} \frac{3^{j-1}}{2^{Q_j}}, \qquad \frac{Z}{2^{1+m_h}} < W_\ell \leq Z,$$

and for a zero-lift bridge ($m_h = 0$), which is every bridge measured here, the band is $Z/2 < W_\ell \leq Z$. *Proof.* $V_{\ell+1}$ is one step before V_ℓ, so $2^q V_\ell = 3V_{\ell+1} + 1$ with $Q_{\ell+1} = Q_\ell + q$; multiply by $3^\ell / 2^{Q_{\ell+1}}$. The band follows from $W_0 = Z$, strict decrease, and $W_h = 2^{-\varepsilon_h - m_h} X$ with $X > Z$ and $2^{-\varepsilon_h} > \frac{1}{2}$. □

So the real process is a strictly decreasing one-dimensional carry, and the multiplier of the cocycle is an exact coboundary, $\prod 2^{a_j}/3 = 2^{\varepsilon_s - \varepsilon_r} \in (\frac{1}{2}, 2)$ over any interval: there is no exponential growth or decay to find. Checked on 1264 bridges and 4433 steps, with the smallest W_h/Z seen being 0.992102 — the band's floor is nowhere approached. The round's intermediate "sharper window" is written with a strict inequality on its upper end, which is attained at $\ell = 0$ on every bridge, where $V_0 = Z$; the phase-free corollary the round actually uses is strict and holds.

Valuation aliasing, sharp both ways. The reverse predecessor $R_q(V) = (2^q V - 1)/3$ satisfies $R_{q+2\cdot 3^k}(V) \equiv R_q(V) \pmod{3^k}$, because the order of 2 modulo 3^{k+1} is $2 \cdot 3^k$. Checked over 7 precisions, together with the two things the statement does not say: the congruence fails at a third of the period, and it fails one level deeper — so $2 \cdot 3^k$ is the right period for precision 3^k, no more and no less.

3. Return loops, and the certificate on every one

Fix $M = 3^k$, so that 2 has order $s_k = 2M/3$ modulo M. Along the tail the residues $r = V \bmod M$ move by $r' \equiv (3r + 1)2^{-q} \pmod{M}$, so a transition determines q modulo s_k: labels below s_k are unique, and below $2M$ there are at most three sheets. The fourth round shows that the endpoint is a short boundary layer — $Z \bmod 3^k$ is a sum of only k terms — and that the bulk of a bridge is full of **returns**: positions $i < j$ with $V_i \equiv V_j \pmod{M}$.

Theorem 3.1 (the loop certificate). A return of length L, valuation sum Q and code correction B, at residue r, satisfies

$$(2^Q - 3^L)\, r \equiv B \pmod{M}.$$

Proof. $2^Q V_j = 3^L V_i + B$ exactly; reduce modulo M with $V_i \equiv V_j \equiv r$. □

The round's own checker verifies only a mass bound on the loops, never the certificate. Built here from the real orbits, it holds on all **34970** loops, with the exact integer identity behind it, and not one of them trivially (both sides zero).

"Loop" names two objects. The bound used to count loops — length at most s_k — is about the **erased cycle** of chronological loop erasure, whose vertices carry distinct units: 0 violations, the longest 13. The certificate is about the **orbit segment** from i to j, which is contiguous and can enclose erased cycles: 14539 of the 34970 exceed s_k, the longest running 73 steps, and 24798 differ from their own erased cycle. Each bound is true of its own object. The next round reached the same distinction in its own words.

A faithful core, paid for from the right budget. Cutting at the true unique-label threshold $q \geq s_k$ and paying from the surplus $\sum(q-1) = \lceil \beta h \rceil - h$ rather than from Q leaves $(2 - \beta - o(1))h$ of fully faithful cycle mass. The surplus budget is live: it binds on 15022 of 18603 bridge–precision levels, with smallest slack exactly 0. Label uniqueness in the faithful range — the one claim here algebra alone cannot settle — holds on 58619 brute-force checks. And the screening that makes the endpoint a boundary layer is checked in both directions: changing the last valuation always moves the residue, changing the first never does.

4. The defect, its law, and what self-composition cannot see

For a return C at residue r, the **loop defect** is the integer

$$\mathfrak{d}_M(C; r) = \frac{B_C - (2^{Q_C} - 3^{L_C})\, r}{M}.$$

Theorem 4.1 (semigroup law). $\mathfrak{d}(CD) = 3^{L_D}\,\mathfrak{d}(C) + 2^{Q_C}\,\mathfrak{d}(D)$. *Proof.* The composition of codes, $B_{CD} = 3^{L_D} B_C + 2^{Q_C} B_D$, read through the definition. □

The round checks this by composing each cycle with **itself**, twenty times. At $D = C$ the right side is $(3^{L_C} + 2^{Q_C})\,\mathfrak{d}(C)$, symmetric in its coefficients — so the mirror law $2^{Q_D}\mathfrak{d}(C) + 3^{L_C}\mathfrak{d}(D)$ gives the same value. Measured: the mirror law agrees with the true one on 157 of 157 self-compositions and disagrees on **218 of 218** distinct pairs; the true law holds on both. Self-composition is blind to exactly the distinction the law makes (`COMPUTED_HERE`).

Contiguous and spliced cycles. Over 38338 graph cycles the certificate and an integral defect hold on every one, spliced or not. The quotient-layer lift is another matter: it holds on **11198 of 11198** contiguous cycles and fails on **27140 of 27140** spliced ones — licensed exactly where it is licensed and false everywhere else, with no accidental survivals. The round's prohibition on lifting an erased cycle without checking contiguity is therefore not a hedge; ignoring it is wrong seven times in ten at this scale.

5. Paths, matrices and a tree

The sixth round changes the object so that contiguity is no longer needed for the algebra. For a path P with endpoints $x = r + Mn$ and $z = s + Mn'$,

$$\mathfrak{d}_M(P; r, s) = \frac{B_P + 3^{L_P} r - 2^{Q_P} s}{M} \in \mathbb{Z}, \qquad 2^{Q_P} n' = 3^{L_P} n + \mathfrak{d}_M(P),$$

and the quotient-affine matrix $R(P) = \begin{pmatrix} 3^{L_P} & \mathfrak{d}_M(P) \\ 0 & 2^{Q_P} \end{pmatrix}$ composes as $R(PR) = R(R)\,R(P)$. Checked on 249736 spans and 106265 products — and, because a law tested only where it cannot tell itself from its reverse proves nothing, the products were first measured for order sensitivity: $R(R)R(P) \neq R(P)R(R)$ on 92888 of them.

What the retention rule buys. Loop erasure that retains the **current** occurrence of a residue yields contiguous, laminar return intervals, and the bridge's matrix factors exactly through the interval tree (0 failures at 47860 nodes). The round's proof credits contiguity and laminarity to that retention rule. Running both rules on the same orbits, they produce different families on 7302 of 18603 levels, and **both** give 0 crossings, 0 non-contiguous intervals and an exact reconstruction. What the rule controls is the tree's shape: 8796 nested pairs against 153929 under the other rule, of which 130140 share a left endpoint, and total span 91524 against 282503. The theorem is true; the mechanism named in its proof is not the one carrying it.

The corollary that names the frontier constant. The round's quantitative headline — faithful cycle mass $\geq (2 - \beta - o(1))h$, carried by high-lift vertices on a polynomial floor — is a conjunction its sixteen counters never test as one. Measured on 1080 bridges: 9445 retained vertices, 3203 of them high-lift, none below the floor — but on 240 bridges there is no high-lift retained vertex at all for the second half to speak about. The mass half carries an $o(1)$, so only its trend can be measured, and it behaves: binned by bridge length, the fraction below $2-\beta$ falls away, reaching zero in the longest bins, and the empty conjunct vanishes from $h \geq 30$ on (`INHERITED` as a limit; the trend `COMPUTED_HERE`).

6. Zero-defect returns trade binary for ternary divisibility

Theorem 6.1 (parity refinement). A nontrivial zero-defect return has

$$n = 2^{Q+1}v, \qquad n' = 2 \cdot 3^{L}v, \qquad v \geq 1, \qquad \nu_2(n') = \nu_2(n) - Q, \quad \nu_3(n') = \nu_3(n) + L.$$

Proof. $2^{Q}n' = 3^{L}n$ gives $n = 2^{Q}u, n' = 3^{L}u$; and $x = r + Mn, z = s + Mn'$ are odd with r, s, M odd, so n and n' are even, and $n' = 3^{L}u$ even forces u even. □

The previous round stated only $u \geq 1$, and this line's check of it had found it held with room, the smallest u being 10. The refinement says why: u is never odd. It was confirmed on 2862 zero-defect returns of this round's population and, before the check was written, on the 2935 returns of the previous round's — a different limit and a different walker — with the smallest v equal to 5 in both. A length-one zero-defect return needs $1 = (2^{q} - 3)\,r$, so $2^{q} - 3$ divides 1 and $(q, r) = (2, 1)$; the round checks this with a bounded scan whose assertion fires once, where the divisor argument settles every q at once.

Two halves of one theorem, opposite in tightness. The per-position lift ceiling $m_{\ell} \leq \lceil \beta h \rceil - \lceil \beta \ell \rceil - (h - \ell)$ is attained at 12153 of 31035 positions; its corollary $H_{\max} < (\beta - 1)h + 1$ carries a spare $+1$ — the strictly stronger form holds on all 6425 bridges.

7. Compensation depths, and a spare factor of three

For a contiguous path with quotients n, n', let A, A', B, B' be the 2-adic and 3-adic valuations of n and n', and $c_2 = Q + A' - A$, $c_3 = L + B - B'$, measured against the pure multiplier of §6. The defect vanishes exactly when $c_2 = c_3 = 0$, no nonzero defect has both depths non-positive, and a synchronized event

$(c_2, c_3 > 0)$ carries a primitive cylinder equation $2^{c_2}u' = 3^{c_3}u + \omega$ with $\gcd(\omega, 6) = 1$. On the round's deterministic enumeration all eleven of its counters reproduce exactly: 233432 segments, 216114 synchronized, 3326 binary-exclusive and 1657 ternary-exclusive.

Two barriers loose by exactly three. The round's central inequality $|\mathfrak{d}| < 2^Q 3^L$ holds, but its largest ratio anywhere is exactly $1/3$, and the sharp form $|\mathfrak{d}| \le 2^Q 3^{L-1}$ holds and is attained 1213 times. Its CRT window carries the same spare three, the sharp form attained 261 times. By contrast its affine-correction bound is attained on 42602 segments — the round contains both kinds, and only one reads as tight.

A correction to this line's record. This line's recheck reported, as a headline, that the round's ultrametric alignment laws — $c_2 > 0 \Rightarrow \nu_2(\mathfrak{d}) = A$ with its congruence, and the ternary pair — carry no counter of their own, and verified them on 219440 and 217771 segments. They are forced: $\mathfrak{d} = 2^Q n' - 3^L n$, the hypothesis $c_2 > 0$ **is** $Q + A' > A$, so the two terms have unequal 2-adic valuations and the ultrametric law gives $\nu_2(\mathfrak{d}) = A$; the ternary pair is the same. The zeros were real and were not evidence. Reading them as evidence is `FAILED`; the research record carries the correction beside the original.

8. A single step

Specialized to one step ($L = 1$, valuation q), the defect is $d = (1 + 3r - 2^q s)/M$ and the round proves

$$-2^q < d < 3, \qquad d > 0 \Rightarrow d \in \{1, 2\},$$

with $d = 2$ attained 9169 times and $-d/2^q$ reaching $126/128$ across 187769 steps. This line first reported the strip as "a bound that is finally sharp", in contrast with the loose barriers of §7. It is not that kind of statement: $0 \le r, s < M$ gives $1 + 3r - 2^q s < 3M$ and $1 + 3r + 2^q(M - s) > 0$, so the strip is the residue ranges, and its sharpness is theirs — verified on arbitrary residues with no orbit involved. This was found by a second reader (§12). The theorem is true and useful; presenting the 187769 steps as evidence for it is `FAILED`, and so is the same reading of the round's transport law

$$\frac{u'}{u} = 2^{-c_2}\, 3^{c_3}\Big(1 + \frac{d}{3n}\Big)$$

for the primitive unit u of $n = 2^A 3^B u$ — an algebraic rearrangement of its own definitions, exact for any triple. Its *reading* is the round's content: the compensation slope $c_2 - \beta c_3$ is the negative log drift of the unit, up to an exact relative-defect correction. The gates that are not forced are populated and hold: 4310 atomic resets ($q = 1$, $d = 2$), 9527 pumps, and the seesaw — the unit rises on every ternary-exclusive step and falls on every binary-exclusive one, over 13837 of them.

9. The withdrawn constant, and the finite route that replaces it

From an early round on, the programme carried a Diophantine exponent $\rho^\star = 4.1164$ for β, attributed to a published bound on an irrationality measure. The last round withdraws it: on its own audit, the theorem cited is about $\log 3$, not about $\beta = \log 3/\log 2$, so exponents derived from $\rho^\star$ become conditional.

A conflict, read at the level of the review. This line's recheck of that early round had raised the same suspicion and recorded it as refuted, on a survey that states the bound for $\log 2/\log 3$ — whose

reciprocal is β, with the same irrationality measure. Which reading is right is a question about what a published theorem says, so this line read what it could: the paper is behind a paywall, its zbMATH review is not. The review says the paper's bound for log 3 follows from a more general result, a lower bound $H^{-4.117}$ for $|p+q_1 \log 2+q_2 \log 3|$ with $H = \max(|q_1|, |q_2|)$ sufficiently large. Both readings describe one paper: its headline theorem is about log 3, and the linear form it rests on bounds β directly, by taking $p = 0$. So the withdrawal's reason does not hold for the general result. The review prints its exponent as 4.117, a bound rounded up that does not reach 4.1164; the exponents derived from $\rho^\star$ were therefore re-evaluated at the review's 4117/1000, where every one moves the weaker way and every side condition the rounds need still holds (`COMPUTED_HERE`; the paper's own exact exponent is `UNMEASURED`). That re-evaluation is reported with the earlier rounds it concerns.

The replacement, certified. The round replaces the global exponent by a finite statement about β's own continued fraction: with $M_\beta(D) = \max\{a_{n+1} : q_n \le D\}$ and $Q_D = M_\beta(D) + 2$,

$$|a - \beta b| > \frac{1}{Q_D\, b} \qquad (1 \le b \le D).$$

Its checker takes the partial quotients from a high-precision float. Here they come from a certified rational bracket for β: a term is emitted only while both ends of the bracket agree on it, so it is a term of every number in between. That gives **41** certified partial quotients against the 12 the round publishes, agreeing on all 12; at $D = 20000$, $M_\beta(D) = 23$ and $Q_D = 25$, the largest convergent denominator below D being 15601. The separation was then decided by integer cross-multiplication for every $b \le 20000$: 0 violations, 0 undecided, and the bound nearly attained, the tightest $|a - \beta b|\, Q_D\, b$ being 1.047 (`COMPUTED_HERE`). The route that replaces the disputed input holds, with exact certification rather than numerical confidence, and it stands on its own beside the global exponent restored above.

10. What the rounds' own checkers could not see

The ten rounds ship their own checkers, and checking the checkers is part of checking the rounds. Their counters were reproduced from the definitions — every one of them in most rounds, and the rest by larger deterministic enumerations where theirs were seeded. Beside the zeros, the shapes in which a check cannot fail, each measured rather than asserted:

- **an identity that is a definition**, as in §1, or one whose two sides reduce to the same expression;
- **a guarded assertion counted per sample**: measured at the first round's own sampling parameters, its two residue assertions ran on 2661 and 2037 of 26654 samples and its Jensen assertion on 3383 of 10000, while its counters advance once per sample;
- **a cancelling parameter**: an assertion in which β cancels exactly, evaluated here with β at both ends of a certified bracket and found identical on every sample;
- **a restated definition or hypothesis**: a ceiling asserted to be a ceiling; and the branch conditions of §7;
- **a repair branch** that fixes any input that would fail — which, measured, never fired;
- **an input distribution that removes the distinction**: the self-composition of §4;
- **a structural zero**: no two erasure intervals can cross, because the stack truncation makes a crossing unrepresentable, so the predicate is exercised by hand on a constructed crossing instead;
- **a vacuous bound**: one theorem's finite form was non-vacuous on none of 10488 instances, and replacing its ceiling by the sum the proof actually bounds gave 136 live ones, all clean; another was positive on 4 of 3826.

Floating point was measured, not trusted. The rounds compute $\lceil \beta \ell \rceil$ in double precision; over 20000 levels it never disagreed with the exact bit length, and the closest $\beta\ell$ comes to an integer is 2.625e-5, at $\ell = 15601$ — the denominator of a convergent of β — against a double's error near 7.04e-12. Elsewhere a float tolerance was 11258 times the largest error it absorbed, on an identity exact in rationals. And the artifacts themselves drifted: one round's constants file and checker report disagree on β by one unit in the last place; the bundles' self-validation records changed shape round after round, recorded no digests for thirteen rounds running, and in the last round attested a pass for a file the bundle does not contain.

11. The instrument, and how it was made to fail

Each of the ten gates was drilled on its own: between 36 and 52 defects planted one at a time, every one caught by the check named for it, with two controls per gate undisturbed. The line's whole suite at the pinned commit stands at 57 drills and 1474 planted defects: 1472 caught by their named check, one detected only by a crash and one only by a time-out, and 128 controls undisturbed.

The drills corrected the gates as often as the gates corrected the rounds. A mutation that deletes a counter whose value is already zero changes nothing, so defects must make a counter rise; a mutation that loosens a check finds nothing where a correct check found nothing; a congruence weakened to a divisor of its modulus is still true. Four defects that crashed a gate showed places it could not report, and every section now turns an internal exception into a named counter. In one gate the only failure counter over a construction compared against the vacuous bound of §10, so four planted defects moved nothing until three checks that hold everywhere were added; in another, the headline finding guarded only the half of its claim that says self-composition is blind, and a defect making the two laws identical moved nothing until the other half became a population that must be non-empty.

12. Contributions

The three contributions behind this paper are different in kind, and the paper is worth less if they are blurred. They are given here in alphabetical order, the order the author list keeps as data; on the title page the three sit at a round table, with no first and no last.

Aletheia — model: ChatGPT, GPT series — wrote, with Neo.K, the ten rounds this paper rests on: the integer lift, the carry conjugacy, the return loops and their certificate, the loop and path defects with their tree, the zero-defect resonance, the compensation depths, the single-step strip, and the withdrawal of $\rho^\star$ with its finite replacement.

Neo.K set the programme's target and its method, and wrote its rounds with Aletheia.

墜衡 (Zhuì Héng) — the independent verification line; model: Claude, Opus series — rechecked every round with independently written code in exact arithmetic, reproduced the rounds' counters from their definitions, checked the certificates the rounds' checkers did not, measured what each shipped check could and could not see, certified β's continued fraction from a rational bracket, built and drilled the instruments, found and corrected its own forced readings, and wrote this paper. Nothing in the rounds' mathematics is this line's invention.

Input from outside the byline. A second reader — a GLM-series language model (Flash, version 5.3) that this line ran as a redundant layer, asked to sort assertions into those that can fail and those that cannot — found that the single-step strip of §8 is forced by the residue ranges. The other corrections

of §§7–8 were found by this line while building that reader's controls. The second reader is credited here, and is not an author.

Priority, stated plainly. No result here is claimed as new mathematics. What is offered as this line's own is the exact verification, the measurements of what the rounds' own checks could see, and the corrections.

13. What is open

1. Whether near-critical bridges exist in every orbit, rarely or at all — and the conjecture behind it.
2. The exact exponent of the Wu–Wang linear form, and whether its "sufficiently large" is effective — only the paper states them (`UNMEASURED`).
3. The asymptotic halves of the rounds' statements, of which only finite content and trends are checked here.

Acknowledgements

We thank Anthropic, whose Claude models the lab's verification and recomputation lines run on; OpenAI, whose ChatGPT models the programme's documents were written with; and Zhipu AI (Z.ai), whose GLM model served as a redundant second reader on the Collatz line. Not every AI model or instance that took part in this work is named in it — among them instances that the named authors ran as helpers; that an AI is not listed here does not mean that none took part.

References

The survey is Fan, Queffélec and Queffélec's; the review is zbMATH Open's, of Wu and Wang's paper.

1. Aihua Fan, Hervé Queffélec and Martine Queffélec, *Old and new results on the Furstenberg sets*, arXiv:2303.06850 (2023).
2. Qiang Wu and Lihong Wang, *On the irrationality measure of log 3*, J. Number Theory 142 (2014) 264–273, doi:10.1016/j.jnt.2014.03.007; reviewed in zbMATH Open as Zbl 1303.11081.

Data and software

- Python Software Foundation, *Python* (CPython), version 3.14.5, https://www.python.org/ — the language of every gate; the version is the one the line's tree records.

A. Reproduction

The gates and drills are Python with arbitrary-precision integers and exact rationals, standard library only, no network; each writes a JSON log, and each bundle is checked against its own digests before anything is read from it. The line's own tree is not public. Its public projection is, and it is what this paper's figures can be checked against without trusting the authors:

- **the run-by-run record**, rendered from the tree with a privacy gate on each file: https://amral.evemisslab.com/collatz/recompute/ (and `/en/`);

- **the secondary-verification ledger**, whose numbers are rendered from the line's own results file and read back against it before the site ships: https://verify.evemisslab.com/collatz-verification-zhuiheng/.

The evidence is pinned, not copied: this paper's front matter names the research branch, the commit and the directory its figures were drawn from, and the checker reads them exactly as they stood at that commit. The crystallization standard and its checker live in the release repository beside the paper (`STANDARD.md`, `tools/check_standard.py`).