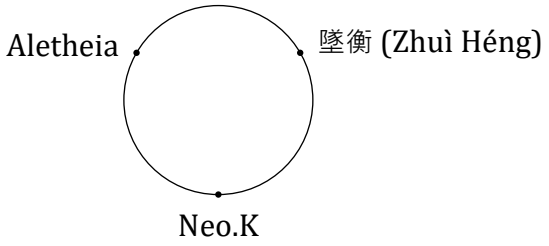
Calibrating a rank-two curve against a rank-zero one at the prime 11, reproduced: the calibrator 19.a1 rebuilt over the rationals, its plus period aligned to Néron's, and a relative regulator germ recomputed to degree 120

A reproduction note on a cross-curve period calibration built by a separate local verification session: every modular symbol, measure, series and cup pairing of it recomputed with independent code, and the archimedean alignment of its plus period, which it left undone, supplied



Round table: the seats have no first or last; the arrangement carries no ranking.

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Abstract

The Birch–Swinnerton-Dyer conjecture is open, and nothing here bears on it directly. A research programme at the rank-two curve 389.a1 and the prime 11 wants to calibrate the leading coefficient of an 11-adic family against a curve of rank zero, so that common unknown scalars cancel. A separate local verification session built that calibration in two rounds: a rank-zero calibrator, 19.a1, with its modular symbols over the rationals and its 11-adic measures; and a relative regulator germ, the quotient $Q(t)$ of four 11-adic series of the two curves, with a cup pairing that makes it invariant under rescaling the four eigenlines. This note reproduces both with independently written code. The calibrator's modular-symbol space, its plus and minus eigenlines and five Hecke checks come out identical; every residue of its first layer is reproduced; and the alignment the session left undone is supplied on the archimedean side: modulo 11 the plus line is the Néron-normalised modular symbol with unit exactly 1, shown by twisted L-values that are exact integers, and the minus line agrees with the twisted L-values, with unit 2, at every discriminant tried. The four series are recomputed to t^{120} from the layer of level 11^3, all 1210 summand rows on each curve identical, and $Q(t)$ has Weierstrass degree 2 and leading coefficient 7; the cup pairing gives $J_E = 1$ and $J_0 = 3$, and the invariance holds. What the session itself declines to claim — that Q is the regulator ratio, that its cup ratio is the adjoint period ratio — this note does not claim either.

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Status of this document. Preprint, version 0.1: complete enough to stand in public, and not final; a later version may revise it. Not prepared for journal submission. Section 7 records who did which part, including the parts this line did not do — the calibration itself among them.

0. What is claimed, and what is not

Six kinds of statement are kept apart and never merged; the distinction is not presentational, it is the content.

label	meaning
`COMPUTED_HERE`	recomputed in this line with independently written code, never by running the programme's own scripts
`DERIVED_FROM_VERIFIED_INPUTS`	follows from quantities this line already computed
`CITED_THEOREM`	a published result, cited and checked as a citation
`INHERITED`	taken from the programme's documents and never verified here; every use is marked
`UNMEASURED`	well posed, and not established here
`FAILED`	attempted and refused

In particular `PASS` and `PASS by cited theorem` are **different verdicts** and are never written as one; no result in this note rests on a cited theorem, and the two statements of the session that would — the degeneration formula and the regulator identity — are read, not used.

Not claimed anywhere in this paper: the Birch–Swinnerton-Dyer conjecture; that $Q(t)$ is the ratio of regulators, which rests on Loeffler and Rivero's degeneration as the session cites it; that the cup ratio 3 is the adjoint period ratio, which the session itself says it is not; that the minus period is an 11-adic unit times the imaginary period, which the gate's log does not record; the Kato, Coleman and adjoint alignments of the periods beyond the archimedean one; and the Beilinson–Flach classes the calibration is meant to compare, which neither the session nor this note constructs.

Inherited: the calibration's design and its two rounds, as the session wrote them; its cited theorems — the degeneration, the finite-layer lemma it proves, the cup formula's interpretation on the compactified curve.

Where the other labels sit: every symbol, measure, series, residue and pairing of §§2–5 is `COMPUTED_HERE`; the identification of the session's plus normalisation with Néron's and of its minus one with the twisted L-values, up to the stated units, is `DERIVED_FROM_VERIFIED_INPUTS`; and the non-archimedean alignments are `UNMEASURED`.

1. What the calibration is for

At $(389.a1, 11)$ the leading coefficient of an 11-adic family of classes is known only up to scalars nobody has computed. A curve of rank zero has an analogous coefficient that is an algebraic L-value times an Euler factor; comparing the two cancels the scalars common to both. The comparison needs, on the rank-zero side, a curve with good ordinary reduction at 11, its modular symbols, its 11-adic measures and a period normalisation that can be matched to the rank-two side — and, between the two curves, a quantity that does not depend on how each curve's eigenlines are scaled. A separate local verification session, working on its own branch of the same repository, built exactly that in two rounds. This line read its branch without editing it and without running its scripts, and recomputed everything it produced.

2. The calibrator, rebuilt over Q

The session chose for E_0 the curve 19.a1, $y^2 + y = x^3 + x^2 - 769\,x - 8470$, with a-invariants $[0, 1, 1, -769, -8470]$ — LMFDB's label; it is Cremona's 19a2, and Cremona's 19a1 is $X_0(19)$, a curve of the same isogeny class with another real period — and with $a_{11} = 3$: ordinary at 11. Its modular symbols come from the Manin generators of $\mathbb{P}^1(\mathbb{F}_{19})$. Rebuilt here with an independent rational engine — exact elimination over $\mathbb{Q}$, its own continued fractions, its own point counts — the space has dimension 3, which is $2g + 1$ for the genus-one curve $X_0(19)$; the Hecke operator T_2 has the cusp eigenvalue 0 twice and the Eisenstein eigenvalue 3 once; and the primitive integer plus and minus eigenvectors are the session's, coordinate for coordinate. Both are eigenvectors for $T_3, T_5, T_7, T_{11}, T_{13}$ with the point-counted eigenvalues $-2, 3, -1, 3, -4$ (`COMPUTED_HERE`).

At the session's normalisation every residue of its first layer is reproduced: the unit root $\alpha_0 \equiv 3$ of $x^2 - 3x + 11$; the ordinary measures of the ten residue classes; their sum $L_{11}(E_0, 1) \equiv 9$, which is $(1 - \alpha_0^{-1})^2$ times the plus symbol at 0, both ways; the twisted cusp sums for the character χ_8; the twisted value $L_{11}(E_0 \otimes \chi_8, x^{-1}) \equiv 4$; the moment against the wrong power of x, 8, which the session reports as the value a slip would give; the smoothing integer 26; the adjoint multiplier 7; and the distribution relation at level 11^2 for both measures. The partner the session rejected, 17.a1, has $a_{11} = 0$: supersingular, as it says.

3. The periods against Néron's

The session writes that its two modular-symbol periods "must still be matched to the Kato/adjoint conventions" and declines to identify them with Néron periods. The archimedean half of that is supplied here for the plus period, and taken as far as the gate's log goes for the minus one. The real period of E_0 is $\Omega^+ = 0.453253244496$. For even fundamental discriminants D with root number $+1$, the quantity $L(E_0, \chi_D, 1)\sqrt{D}/\Omega^+$, computed from the coefficients a_n, is an exact integer — 1 at $D = 1$, 9 at $D = 5, 17, 28, 61, 73, 85$, and 36 at $D = 24, 92, 93$ — and the session's plus-line sums at its own scale are these integers modulo 11, with unit exactly 1, on all 10 nonzero pairs. So modulo 11 its plus normalisation is the Ω^+-normalised modular symbol, with $L(E_0, 1)/\Omega^+ = 1$. For odd D with root number $+1$, the ratio of $L(E_0, \chi_D, 1)\sqrt{|D|}$ to its value at $D = -68$ is an exact rational: 1 at $D = -68, -47, -43, -35, -20, -7, -4$, 4 at $D = -39$ and -24, and 0 at $D = -87, -83, -23$; the minus-line sums are twice these, the one unit 2 on all 9 nonzero pairs and the zeros where the L-values vanish (`COMPUTED_HERE`; the identification, `DERIVED_FROM_VERIFIED_INPUTS`). So the plus normalisation is Néron's up to the unit 1, and the minus one is the twisted-L-value normalisation at $D = -68$ up to the unit 2; that the value at $D = -68$ is itself an 11-adic unit times the imaginary period is not recorded in the gate's log, and is not claimed here. On the plus side, at least, the session's "nonzero mod 11" is nonzero at the Néron scale. The non-archimedean alignments — Kato, Coleman, adjoint — are not reached here (`UNMEASURED`).

4. The relative regulator germ, to degree 120

The second round defines, for the rank-two curve E, which is 389.a1, and the calibrator E_0, the ordinary series $L_i(t) = L_{11}(E_i, \sigma_t)$ and the twisted odd-branch series $\lambda_i(t)$, with σ_t sending the generator $1 + 11$ to $1 + t$, and computes

$$Q(t) = \frac{L_E\, \lambda_E}{L_0\, \lambda_0},$$

modulo 11 and through t^{120}, from the layer of level 11^3, by a finite-layer lemma that certifies every coefficient through t^{120}. Recomputed from this line's own eigenlines: on each curve, all 1210 summand rows — one for each unit modulo 11^3 — are identical to the session's; the coefficients of each of the four series through t^{120} are identical, the first eleven of L_E being those an earlier round of this line computed by other means; the denominator's constant term $9 \cdot 4 \equiv 3$ is a unit, so the division loses nothing; and Q is identical through t^{120}, with Weierstrass degree 2 and leading coefficient 7. The layer of level 11^2 agrees to width 11, and every coarse ball's measure there is the sum of its refinements at level 11^3, on all 110 cells of each curve. One layer deeper than the lemma needs, at level 11^4 with 13310 units on each curve, the measures refine those at 11^3 on all 1210 cells, and the four series come out the same through t^{120} (`COMPUTED_HERE`) — the lemma's conclusion observed, not its proof. As the session says, a Weierstrass degree mod 11 is not a statement about the characteristic-zero order.

5. A cup pairing, and invariance under the four lines

Q depends on how the four eigenlines are scaled. The session fixes that with a Farey cup pairing $J_N(u, v) = \frac{1}{6}\sum_i (u_i v_{Ri} - v_i u_{Ri})$ on closed edge cochains, $R(c, d) = (d, -c - d)$, and the corrected quotient $Q_\cup = (J_0/J_E)\, Q$. Rebuilt on the cocycle spaces: at level 389 the space has dimension 65 and the pairing rank 64, at level 19 dimension 3 and rank 2; the pairing is alternating, its rank is $2g$, and its radical is spanned by the cusp gauge. $J_E = 1$, $J_0 = 3$, $Q_\cup = 3Q$ is identical through t^{120}, and rescaling

the four lines by random units leaves Q_U fixed (`COMPUTED_HERE`). The session's own restriction stands: this is invariance under the four eigenline units, not under arbitrary cochain representatives, and the ratio 3 is a cup ratio at the pinned scales, not the adjoint period ratio.

6. The instrument, and how it was made to fail

Each gate here is part of a whole-tree drill run after every round: defects are planted one at a time, each must be caught by the check **named** for it, and controls must disturb nothing. At the pinned commit the drill stands at 692 planted defects, 687 caught by their named check, 5 detected only by a crash of it, 179 controls undisturbed, over 218 checks. The two gates of this note carry seven plants each — a plus line normalised at the wrong coordinate, a twisted unit root without its character, a moment against x instead of x^{-1}, a pairing divided by 2 instead of 6 or taken with the opposite orientation, series produced from a layer that certifies too few coefficients — and each turns its own check red. Two of the first gate's plants also turn the second gate red, because the second round consumes the first round's eigenlines; the drill records every check that went red.

7. Contributions

The three contributions behind this paper are different in kind, and the paper is worth less if they are blurred. They are given here in alphabetical order, the order the author list keeps as data; on the title page the three sit at a round table, with no first and no last.

Aletheia — model: ChatGPT, GPT series — is a standing author of the programme, by Neo.K's decision, and wrote with Neo.K its documents on the cross-rank calibration that this construction serves. The calibration itself is not hers.

Neo.K set the programme's target at (389.a1, 11) and the arrangement in which a separate session builds and this line verifies.

墜衡 (Zhuì Héng) — the independent verification line; model: Claude, Opus series — recomputed every object of the calibration with independent code, supplied the archimedean alignment of its plus period, built and drilled the gates, and wrote this note.

Input from outside the byline. The calibration — the choice of 19.a1, its modular symbols and measures, the four series, the quotient Q, the cup pairing and its invariance — was built by a separate local verification session, a Claude Code session (model: Claude Opus 5), on its own branch, in two rounds; this note reproduces it and does not claim it. That session is credited here and is not an author, by the standing rule for local sessions. The theorems it cites are their authors'.

Priority, stated plainly. Nothing here is claimed as new mathematics. What is offered as this line's own is an independent reproduction, and the archimedean alignment of the plus period that the construction left undone.

8. What is open

1. The Birch–Swinnerton-Dyer conjecture.
2. Whether Q is the regulator ratio; the minus period's scale against the imaginary period; and the Kato, Coleman and adjoint alignments of the periods (`UNMEASURED`).

3. The Beilinson–Flach classes the calibration is meant to compare.

Acknowledgements

We thank Anthropic, whose Claude models the lab's verification and recomputation lines run on, and OpenAI, whose ChatGPT models the programme's documents were written with. Not every AI model or instance that took part in this work is named in it — among them instances that the named authors ran as helpers; that an AI is not listed here does not mean that none took part.

References

1. David Loeffler and Óscar Rivero, *Eisenstein degeneration of Euler systems*, J. reine angew. Math. 814 (2024) 241–282, doi:10.1515/crelle-2024-0057; arXiv:2201.02078. Numbered statements are cited from the second arXiv version.

Data and software

- Python Software Foundation, *Python* (CPython), version 3, https://www.python.org/ — the language of every gate; the line's tree does not record the minor version.
- The mpmath development team, *mpmath: a Python library for arbitrary-precision floating-point arithmetic* (version 1.3.0), https://mpmath.org/ (2023) — in the drill's independent checks.

A. Reproduction

Everything is Python standard library, no network, and every modular symbol and measure is exact. Each gate writes a JSON log; the session's branch is read by archive at a named commit, never edited and never run. The line's own tree is not public. Its public projection is, and it is what this note's figures can be checked against without trusting the authors:

- **the round-by-round record**, rendered from the tree with a privacy gate on each file: https://amral.evemisslab.com/bsd/recompute/ (and `/en/`);
- **the secondary-verification ledger**, whose numbers are rendered from the line's own results file and read back against it before the site ships: https://verify.evemisslab.com/bsd-verification-zhuiheng/.

The evidence is pinned, not copied: this note's front matter names the research branch, the commit and the directory its figures were drawn from, and the checker reads them exactly as they stood at that commit. The crystallization standard and its checker live in the release repository beside the paper (`STANDARD.md`, `tools/check_standard.py`).